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AN INVESTIGATION OF THE BURN-IN AND RELATED PROBLEMS

by

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November 1964 ORC 64-32 (RR)

This research has been partially supported by the Office of Naval Research under Contract Nonr-3656(18) with the University of California.

ABSTRACT

Two problems involving the derivation of bounds on distributions with a decreasing failure rate (DFR distributions) are presented. Given that an item has a decreasing failure rate, sharp upper and lower bounds on the burn-in time to achieve a specified mean residual life are derived The bounds rely only on the DFR assumption and a knowledge of the first moment and a percentile of the failure distribution.

An early estimate of the five year survival proportion (commonly called the five year cure rate) is of great interest in assessing the value of a treatment for a mortal disease such as cancer. Assuming that the distribution of time to death is DFR and assuming a knowledge of the mean and a percentile, sharp upper and lower bounds on the survival proportion are obtained.

In addition some bounds on the hazard rate and density of a DFR distribution are given.

1. Introduction. The need for complex electronic equipment in locations where replacement of failed parts is impossible (e.g., ballistic missiles, satellites, etc.) has necessitated the production of very reliable components such as semiconductors. One method which is almost universally used to help achieve high reliability is to pre-age or burn-in the components to eliminate the "sports" or early failures. It has long been known that semiconductors, for example, exhibit infant mortality yet do not wear out (See: Blakemore, Kronson, Von Alven, 1963; Norris, 1963; Von Alven, 1962; and Von Alven, Blakemore, 1961); that is, they exhibit a decreasing failure rate (are DFR). The problem which now presents itself is how long to burn-in the component to achieve a specified reliability. The problem has been answered in the past by assuming a Weibull distribution of time to failure (Von Alven, 1962; Watson, Wells, 1961), for which there seems little statistical validation. However, this assumption is not necessary. It suffices to assume that the distribution is DFR and its first moment and a percentile are known in order to obtain sharp bounds on the residual mean life. It is a simple matter once the bounds are known to determine the minimum burn-in time to achieve a specified residual mean life. This is shown in section 2. Although the burn-in problem is developed in terms of a particular example, burning-in can clearly be used to advantage on any item which exhibits a decreasing failure rate.

A problem which bears some slight similarity to the burn-in problem is that of estimating the five year survival proportion in a

population of cancer patients (also called five year cure rate Berkson, Gage 1952). When a new treatment is tested clinically it is desirable to obtain an estimate, as early as possible, of its effectiveness. One objective indication of this is the five year survival proportion. Berkson and Gage (1952) have estimated a related quantity, the cure proportion, by assuming in the interests of mathematical expediency that the death rate due to cancer is a constant. It is well validated that the death rate from cancer is decreasing with time (Cutler, Axtell, 1963; Berkson, Gage, 1952) and this fact is used together with a knowledge of the mean and a percentile of the distribution of time to death, to obtain sharp upper and lower bounds on the five year survival proportion. This is discussed in section 3.

The most appealing feature of the solutions presented to the preceding two problems is that no assumption of a parametric expression for the probability distribution is made. Such an assumption would be very difficult to verify using the truncated data from semiconductor life tests or the small sample data resulting from a clinical trial. Yet it is possible to obtain a reasonable estimate of the mean life and early percentile for a truncated life test. This information together with the DFR assumption enables us to obtain bounds on the relevant quantities to be evaluated.

Nearly all the work completed to date on bounds for distributions possessing a monotone failure rate has been done by two authors, Barlow and Marshall (see: Barlow, 1963; Barlow and Marshall, 1963, 1964a and 1964b), who have often collaborated.

Barlow and Proschan (1964) also develop many applications of these bounds in the field of reliability theory.

Only the relevant bounds are set forth in sections 2 and 3, while the bounds are derived in section 4. A few additional theorems are added to complete the discussion of bounds for DFR distributions.

Mathematical Preliminaries. If the failure distribution F has density f, then the failure rate $q(t)$ is defined for those values of t for which $F(t) < 1$ by

$$q(t) = \frac{f(t)}{\bar{F}(t)}$$

where $\bar{F}(t) = 1 - F(t)$ and it is assumed that $F(0^-) = 0$. The failure rate is also known as the hazard rate and by actuaries as the "force of mortality".

It can be readily verified that $q(t) = -\frac{d}{dt} \log \bar{F}(t)$, when a density exists. Hence F is DFR (IFR) if $\log \bar{F}(t)$ is convex on $[0,\infty]$ (concave where finite). This fact forms the basis of many of the proofs of section 4.

2. The Burn-in Problem. The only items which are burnt-in to any extent are semiconductors, although burn-in could well be applied more widely. Hence, we use semiconductors to illustrate our discussion of the burn-in problem.

A few remarks will made on the life distribution of a semiconductor since there is still some disagreement as to the actual shape of the distribution. Peck (Von Alven, 1962, Chapter 2) asserts that his data on semiconductors exhibits first an increasing failure rate and then a decreasing failure rate. He supports this by a physical explanation of the cause of the initial IFR . Other authors (i.e., Norris, 1963) maintain the distribution is DFR and the IFR , but this theory is more intuitive than well supported by life test data. The great bulk of work in the life testing of semiconductors supports the theory that there is no wear-out, and the failure rate is always decreasing. In particular, ARINC Research Corporation (Blakemore, Kronson, Von Alven, 1963; Von Alven, Blakemore, 1961) tested 10,300 individual devices with approximately 200,000 separate life test measurements and reported they could not detect any wear-out, but found in almost every case the life distribution was DFR .

The time to burn-in to achieve a specified reliability has been determined in practice by assuming the life distribution is either Weibull or lognormal. This affords a very rapid and simple method for evaluating the burn-in time but in the majority of cases where these distributions are assumed, it is done with little statistical validation.

A non-parametric approach based purely on the DFR assumption obviates this uncertainty of distribution validity and gives sharp bounds which although more conservative, does guarantee achieving the specified reliability since it is valid for a larger class of distributions.

Bounds are set out below on the survival probability and the residual mean life based on the assumption that the distribution is DFR and its mean and a percentile are known.

$$\underline{F \text{ DFR } \mu_1 = 1, \quad F(\xi_p) = p \text{ and } \xi_p \leq 1 \text{ or } 1-p < e^{-\xi_p}}\ ^{*}$$

(2.1)
$$\bar{F}(t) \geq \begin{cases} \alpha e^{-\alpha t}, & 0 \leq t \leq \xi_p \\ e^{-b_1 t}, & t > \xi_p \end{cases}$$

(2.2)
$$\bar{F}(t) \leq \begin{cases} e^{-b_1 t}, & 0 \leq t \leq \xi_p \\ \alpha e^{-\alpha t}, & t > \xi_p \end{cases}$$

where $\bar{F} = 1 - F$. The residual mean life at time t is denoted by μ_t

(2.3)
$$\mu_t = \frac{\int_t^\infty \bar{F}(x)\,dx}{\bar{F}(t)} \geq \begin{cases} (1 - \frac{1}{b_1}) e^{b_1 t} + \frac{1}{b_1}, & t \leq \xi_p \\ \frac{1}{b_2}, & t > \xi_p \end{cases}$$

(2.4)
$$\mu_t = \frac{\int_t^\infty \bar{F}(x)\,dx}{\bar{F}(t)} \leq \frac{1}{\alpha}, \qquad t \leq \xi_p .$$

Where α is the unique solution of

(2.5)
$$\alpha e^{-\alpha \xi_p} = 1 - p$$

and b_1, b_2 satisfy

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*Note that when distribution is DFR, $\xi_p \leq 1$ implies $1 - p \leq e^{-\xi_p}$.

$$1 = \frac{1 - e^{-b_1 \xi_p}}{b_1} + \frac{e^{-b_1 \xi_p}}{b_2}$$

$$1 - p = e^{-b_1 \xi_p}$$

The proofs of (2.1) through (2.4) are given in theorems (4.2) through (4.5) of Section 4.

The most interesting result, the lower bound on the mean residual life, is useful only up to time ξ_p, for after this time the bound is constant and so gives no indication of the effect of increased burn-in. However, even in the light of this, the restriction on the percentile that $\xi_p \leq 1$ or $1 - p < e^{-\xi_p}$ does not appear restrictive in the burn-in problem as it is unlikely that the item would be burnt-in for a time greater than its mean life.

By the use of equation (2.3), the burn-in time to achieve a specified mean residual life has been calculated and is given in tables 1 through 3.

Bounds in terms of the first and second moment have little usefulness since the sharp lower bound on the mean residual life in this case is the value of the mean, thus giving no indication of the benefit gained by burn-in. Bounds on the survival probability for this case have been calculated by Barlow and Marshall (1963).

The burn-in of semiconductors is sometimes carried out at an increased stress to accelerate the ageing, (see Norris, 1963). This may introduce many complications in the prediction of the optimal burn-in time. However, it will be assumed that it is possible to convert from the un-accelerated burn-in time to the accelerated burn-in

time. This could possibly be done with the use of the Arrhenius relation, although Von Alven* states that the Eyring relationship may possibly be better.

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*Personal communication.

TABLE 1

LOWER BOUND ON BURN-IN TIME TO ACHIEVE RESIDUAL MEAN LIFE = 1.25

ξ_p \ p	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.0064	0.0044	0.0033	0.0025	0.0019	0.0014	0.0010
0.02	0.0132	0.0091	0.0066	0.0050	0.0038	0.0028	0.0020
0.03	0.0202	0.0138	0.0100	0.0075	0.0057	0.0042	0.0029
0.04	0.0278	0.0188	0.0136	0.0101	0.0076	0.0057	0.0039
0.05	0.0358	0.0239	0.0172	0.0128	0.0096	0.0071	0.0049
0.06	0.0442	0.0293	0.0207	0.0155	0.0116	0.0086	0.0060
0.07	0.0532	0.0349	0.0248	0.0183	0.0137	0.0101	0.0070
0.08	0.0627	0.0407	0.0287	0.0211	0.0158	0.0116	0.0080
0.09	0.0728	0.0467	0.0328	0.0240	0.0179	0.0131	0.0090
0.10	0.0836	0.0530	0.0370	0.0270	0.0200	0.0147	0.0101
0.20		0.1348	0.0869	0.0606	0.0436	0.0312	0.0210
0.30		0.2781	0.1581	0.1035	0.0716	0.0499	0.0329
0.40			0.2680	0.1603	0.1057	0.0714	0.0455
0.50			0.4619	0.2332	0.1478	0.0961	0.0602
0.60				0.3567	0.2015	0.1251	0.0759
0.70				0.5518	0.2723	0.1593	0.0933
0.80					0.3700	0.2006	0.1127
0.90					0.5145	0.2512	0.1344
1.00					0.7529	0.3150	0.1589

N.B. No entry in the table indicates the burn in time exceeds ξ_p.

TABLE 2

LOWER BOUND ON BURN-IN TIME TO ACHIEVE RESIDUAL MEAN LIFE = 1.50

ε_p \ p	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.0081	0.0059	0.0045	0.0034	0.0025	0.0018
0.02	0.0164	0.0120	0.0090	0.0068	0.0051	0.0035
0.03	0.0250	0.0182	0.0136	0.0103	0.0077	0.0053
0.04	0.0339	0.0246	0.0184	0.0138	0.0103	0.0071
0.05	0.0432	0.0311	0.0232	0.0174	0.0129	0.0090
0.06	0.0527	0.0378	0.0281	0.0211	0.0156	0.0108
0.07	0.0626	0.0447	0.0331	0.0248	0.0183	0.0126
0.08	0.0729	0.0517	0.0381	0.0285	0.0210	0.0145
0.09	0.0836	0.0589	0.0433	0.0323	0.0238	0.0164
0.10	0.0946	0.0664	0.0486	0.0361	0.0266	0.0183
0.20		0.1536	0.1079	0.0780	0.0561	0.0379
0.30		0.2735	0.1820	0.1272	0.0893	0.0592
0.40			0.2773	0.1857	0.1268	0.0822
0.50			0.4050	0.2566	0.1694	0.1072
0.60			0.5864	0.3446	0.2185	0.1346
0.70				0.4570	0.2757	0.1646
0.80				0.6062	0.3431	0.1976
0.90				0.8164	0.4240	0.2342
1.00					0.5230	0.2750

N.B. No entry in the table indicates the burn-in time exceeds [illegible].

TABLE 3

LOWER BOUND ON BURN-IN TIME TO ACHIEVE RESIDUAL MEAN LIFE = 2.00

	0.55	0.60	0.65	0.70	0.80	0.90
0.01	0.0088	0.0076	0.0066	0.0058	0.0043	0.0030
0.02	0.0177	0.0154	0.0134	0.0117	0.0087	0.0061
0.03	0.0268	0.0232	0.0202	0.0176	0.0131	0.0091
0.04	0.0360	0.0312	0.0272	0.0236	0.0175	0.0122
0.05	0.0455	0.0394	0.0342	0.0297	0.0220	0.0153
0.06	0.0551	0.0476	0.0413	0.0358	0.0266	0.0184
0.07	0.0649	0.0560	0.0486	0.0421	0.0311	0.0215
0.08	0.0749	0.0646	0.0559	0.0484	0.0357	0.0247
0.09	0.0851	0.0733	0.0634	0.0548	0.0404	0.0279
0.10	0.0955	0.0821	0.0709	0.0613	0.0451	0.0311
0.20		0.1798	0.1532	0.1309	0.0947	0.0642
0.30		0.2983	0.2502	0.2110	0.1494	0.0997
0.40			0.3663	0.3041	0.2103	0.1378
0.50				0.4141	0.2785	0.1787
0.60				0.5464	0.3554	0.2229
0.70					0.4431	0.2708
0.80					0.5442	0.3228
0.90					0.6623	0.3797
1.00					0.8029	0.4421

N.B. No entry in the table indicates the burn-in time exceeds ξ_p .

3. Bounding the Survival Proportion. A frequently used objective index of the effectiveness of a treatment is the proportion of patients surviving the disease for five years. (See for example, Berkson, Gage, 1952). This will be called the five year survival proportion although it has been generally called by the misleading name of "five year cure rate" by the medical profession. Clearly the word cure is inappropriate as even in so mortal a disease as cancer it is not certain that the patient is cured when he has survived five years.

Berkson and Gage (1952), have discarded the idea of a survival percentage in favour of computing the cure proportion; defining the patient cured when his death rate is the same as the normal mortality rate. Cutler and Axtell (1963) point out that in some cancers this is never achieved and they redefine cure as the achievement of a stabilized death rate. But with both Berkson, et al and Cutler, et al, information is required over a long span of time to estimate the cure proportion.

Although the five year survival proportion does not give as accurate a picture as the cure proportion (where the latter is relevant) it does afford a good indication of the effectiveness of a treatment and can be calculated more simply and at an earlier time than can the cure proportion.

An extension of the work already done in this field would be to predict the five year survival proportion at the end of only say one year of a clinical trial, thus enabling an early assessment of the treatment to be made.

The mathematical model will be simplified by assuming that the probability of death due to normal causes is independent of the probability of death due to cancer. This clearly oversimplifies the issue but Berkson and Gage (1952) maintain that this assumption does give reasonable results. Thus

(3.1) $$\bar{F}(t) = \bar{F}_n(t)\,\bar{F}_c(t)$$

where $\bar{F}(t)$ is the survival probability in time t and $F_n(t)$ and $F_c(t)$ are respectively the probabilities of death from "normal causes" and from cancer in time t. It is assumed that F_n is known from life tables. Also

(3.2) $$q(t) = q_c(t) + q_n(t)$$

where $q(t)$ is the death rate at time t and the subscripts c and n are as in (3.1).

Berkson and Gage (1952) have shown that $q_c(t)$ is decreasing at a rate which is a function of the mean time to death of the untreated patients. The normal mortality rate, $q_n(t)$, is increasing. Thus $q(t)$ is initially decreasing and then increasing.

If the time at which $q(t)$ changes from decreasing to increasing is large compared with 5 years, bounds on $\bar{F}_c$ (5 years) may be estimated by assuming that F is DFR. Thus from a knowledge of the mean and a percentile of F, bounds on $\bar{F}$ (5 years) may be determined by using equations (2.1) and (2.2), and from this $\bar{F}_c$ (5 years) can easily be determined from equation (3.1).

A less accurate but surer method would be to adjust the observations of the time to death to obtain an estimate of the DFR

distribution F_c . Hence by a straightforward application of the DFR bounds, we may obtain bounds on $\bar{F}_c$ (5 years).

Bounds on the failure rate at five years may also be evaluated to give a further indication of the effectiveness of the treatment. These bounds can be estimated by the use of the DFR bounds given below and equation (3.2)

$\underline{F \text{ DFR}, \ \mu_1 = 1, \ F(\xi_p) = p \text{ and } \xi_p \leq 1 \text{ or } 1 - p < e^{-\xi_p}}$

(3.3) $$q(t^+) \geq \alpha$$

where α is defined by the unique solution to (2.5).

(3.4) $$q(t^-) \leq \begin{cases} \infty & t = 0 \\ b_1 & t > 0 \end{cases}$$

See theorems (4.6) and (4.7) for proofs.

4. Bounds on DFR Distributions.

4.1. Bounds Given the Mean and a Percentile have great appeal since good estimates of these two quantities can be obtained from limited life test data. In addition, the desired bounds are all nontrivial which is in contrast to the case where the first two moments μ_1 and μ_2 are given. In this case the lower bound on the mean residual life is trivial and sharp (i.e. is the value μ_1).

The DFR distribution with mean $\mu_1 = 1$ implies an upper bound on the percentile. The lower bound is zero, and both bounds are sharp.

$$(4.1) \qquad 0 \le 1 - p \le \begin{cases} e^{-\xi_p} & , \ \xi_p \le 1 \\ (\xi_p e)^{-1} & , \ \xi_p > 1 \end{cases}$$

This result is proved by Barlow and Marshall (1963).

Two distributions will be defined. Let

$$(4.2) \qquad \bar{J}_{b_1}(x) = \begin{cases} \beta e^{-b_1 x} & , \ x \le \xi_p \\ (1-p)\, e^{-(x-\xi_p)b_2} & , \ x > \xi_p \end{cases}$$

where $\bar{J}_{b_1} = 1 - J_{b_1}$,

and β and b_2 are given by

$$(4.3) \qquad \beta\, e^{-b_1 \xi_p} = 1 - p \ , \quad (0 \le \beta \le 1)$$

and
$$\int_0^\infty \bar{J}_{b_1}(x)\, dx = 1 \ ;$$

Clearly J_{b_1} has mean $\mu_1 = 1$ and p^{th} percentile ξ_p .

Let K_{b_1} be the subclass of J_{b_1} which is DFR . A

lemma will now be proved to facilitate the proof of theorem 4.1.

Lemma 4.1. If the p^{th} percentile ξ_p of K_{b_1} satisfies the conditions (4.1) then the distribution of the class K_{b_1} which gives the maximum value to the mass at the origin is $K_{b_1^*}$, where b_1^* is the minimum value of b_1 such that $b_1 = b_2$; i.e., an exponential on $[0, \infty]$, with possible mass at the origin.

Proof. Clearly the distribution of the class $\mathcal{J}_{b_1}$ which attains maximum mass at the origin is given by $b_1 = 0$; but it is not DFR since it must be log concave. As b_1 increases from zero (thus decreasing β), b_2 increases but less rapidly, and the distribution will first be DFR when $b_1 = b_2$, attaining then the maximum mass at the origin for a distribution K_{b_1}. The equation of this distribution is

(4.4) $$K_{b_1 = b_2}(x) = \alpha e^{-\alpha x}$$

where α is the minimum solution of

(4.5) $$\left.\begin{array}{l} 1 - p = \alpha e^{-\alpha \xi_p} \\ \alpha \leq 1 \end{array}\right\}$$

Note that a solution α is guaranteed by the conditions on the percentile, and that for $\xi_p \leq 1$ the solution to (4.5) is unique, but for $\xi_p \geq 1$ there may be two solutions.||

Theorem 4.1. If F is DFR with mean $\mu_1 = 1$, and p^{th} percentile ξ_p is given and satisfies the condition $1 - p < e^{-\xi_p}$, then

$$\bar{F}(x) \geq \alpha e^{-\alpha x} \quad , \quad x \leq \xi_p$$
$$\bar{F}(x) \leq \alpha e^{-\alpha x} \quad , \quad x \geq \xi_p$$

and α is defined as the unique solution of $\alpha e^{-\alpha\xi_p} = 1 - p$.

Proof. Suppose that the theorem is false. Then by convexity, either (i) or (ii) is true.

(i) $\bar{F}(x) < \alpha e^{-\alpha x}$, for some $x < \xi_p$

(ii) $\bar{F}(x) > \alpha e^{-\alpha x}$, for some $x > \xi_p$ and $\bar{F}(x) \geq \alpha e^{-\alpha x}$ for $x \leq \xi_p$.

Suppose case (i):

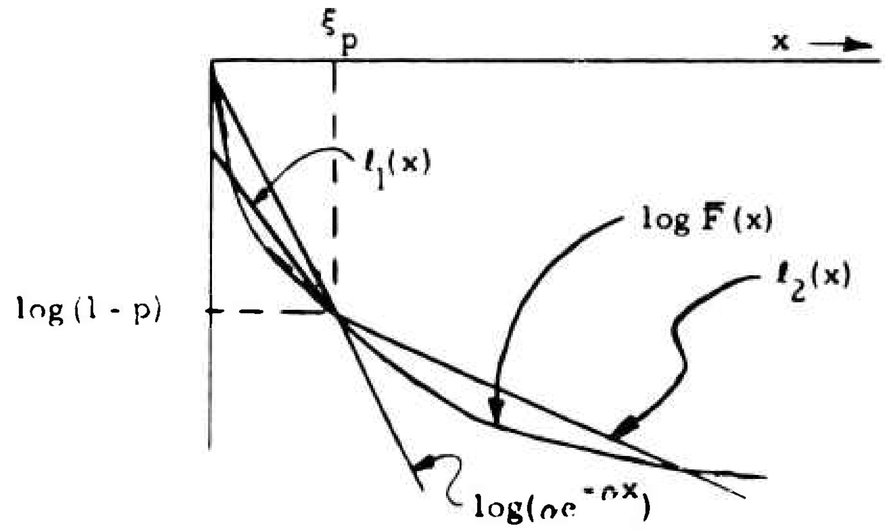


Figure 4.1

By log convexity $\bar{F}(x) > \alpha e^{-\alpha x}$ when $x > \xi_p$. Construct two exponential curves, $e^{-\ell_1(x)}$ and $e^{-\ell_2(x)}$ where $\ell_1(x)$ and $\ell_2(x)$ are linear in x and such that

$$e^{-\ell_1(\xi_p)} = e^{-\ell_2(\xi_p)} = 1 - p$$

$$\int_0^{\xi_p} \left\{ \bar{F}(x) - e^{-\ell_1(x)} \right\} dx = 0$$

$$\int_{\xi_p}^{\infty} \left\{ \bar{F}(x) - e^{-\ell_2(x)} \right\} dx = 0 \ .$$

Now clearly $e^{-\ell_2(x)} > \alpha e^{-\alpha x}$ for $x > \xi_p$ and hence $e^{-\ell_1(x)} < \alpha e^{-\alpha x}$ for $x < \xi_p$. Thus a DFR distribution has been constructed with the same mean and percentile as the exponential $\bar{K}(x) = \alpha e^{-\alpha x}$, but a greater mass at the origin, which by lemma 4.1 is impossible.

Suppose case (ii):

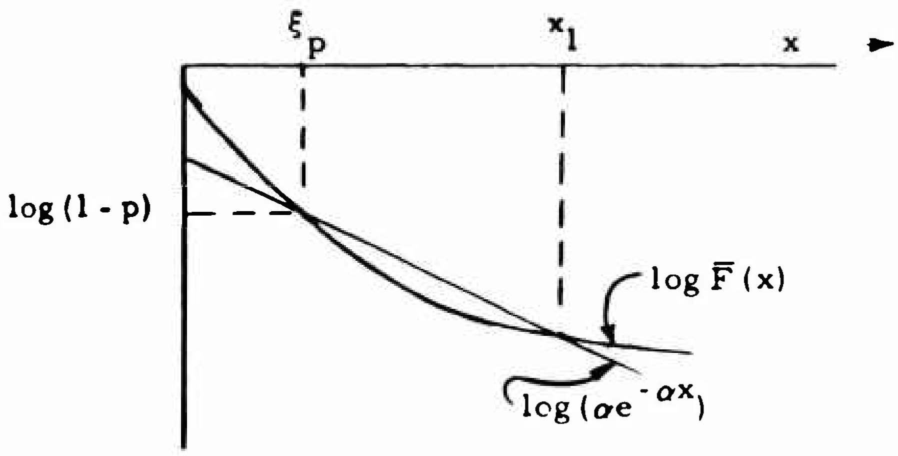


Figure 4.2

By log convexity of DFR distributions there must be a single crossing, say at x_1 , where $x_1 > \xi_p$. By assuming a percentile at x_1 , and noting that the solution α is unique since $1 - p < e^{-\xi_p}$, it can be seen from case (i) that case (ii) is impossible. ||

Note that the condition on the percentile $1 - p < e^{-\xi_p}$ which it was necessary to assume to assure a unique value of α is always satisfied by a DFR distribution for $\xi_p \leq 1$.

It does not appear likely that this restriction will limit the applicability of the bounds as it is hard to imagine estimating a percentile at $\xi_p > 1$ in a distribution. In order to prove some theorems on bounds, a class of DFR distributions $\bar{G}_T$ is posited with mean $\mu_1 = 1$ and p^{th} percentile ξ_p satisfying $1 - p < e^{-\xi_p}$.

$\underline{T < \xi_p}$

$$(4.6) \qquad \bar{G}_T(x) = \begin{cases} e^{-b_1 x} , & 0 \le x \le T \\ e^{-b_2 x + (b_2 - b_1) T} , & x > T \end{cases}$$

where b_1 and b_2 satisfy

$$(4.7) \qquad 1 = \frac{1 - e^{-b_1 T}}{b_1} + \frac{e^{-b_1 T}}{b_2}$$

$$(4.8) \qquad 1 - p = \exp(-b_2 \xi_p + (b_2 - b_1) T)$$

and for this class to be DFR, $b_2 \le b_1$.

$\underline{T \ge \xi_p}$

$$(4.9) \qquad \bar{G}_T(x) = \begin{cases} e^{-b_1 x} , & 0 \le x \le T \\ e^{-b_2 x + (b_2 - b_1) T} , & x > T \end{cases}$$

b_1 and b_2 satisfy (4.7) and

$$(4.10) \qquad 1 - p = e^{-b_1 \xi_p}$$

and $b_2 \le b_1$ for G_T to be DFR.

<u>Lemma 4.2</u> Assuming $1 - p \le e^{-\xi_p}$ for every $T < \xi_p$ there is a solution of (4.7) and (4.8) continuous in T, and for every $T \ge \xi_p$ there is a solution of (4.7) and (4.10) continuous in T.

<u>Proof:</u> $\underline{T \le \xi_p}$: By substituting for b_2 from (4.7) into (4.8) we

obtain

$$1-p = \exp\left\{-b_1T - e^{-b_1T}(\xi_p - T)\left(1+\frac{e^{-b_1T}-1}{b_1}\right)^{-1}\right\}$$

and it is desired to show this always has a solution b_1 . Since the distribution is IFR and $1-p \leq e^{-\xi_p}$, we see $b_1 \geq 1$.

Let

$$h(b_1) = \exp\left\{-b_1T - e^{-b_1T}(\xi_p - T)\left(1+\frac{e^{-b_1T}-1}{b_1}\right)^{-1}\right\} - 1+p .$$

Now $\qquad \lim\limits_{b_1\to 1} h(b_1) = e^{-\xi_p} - 1 + p \geq 0$

$$\lim_{b_1\to\infty} h(b_1) = -1+p < 0$$

Thus as the function is clearly continuous in b_1 , there exists a solution such that $h(b_1) = 0$.

Since $b_1 \geq 1$ and the mean $\mu_1 = 1$, $\bar{G}_T$ must cross e^{-x} once from below and thus $b_2 \leq b_1$.

$\underline{T > \xi_p}$: From (4.10) it can be seen that there is always a solution for b_1 .

i.e. $\qquad b_1 = -\frac{1}{\xi_p}\log(1-p)$.

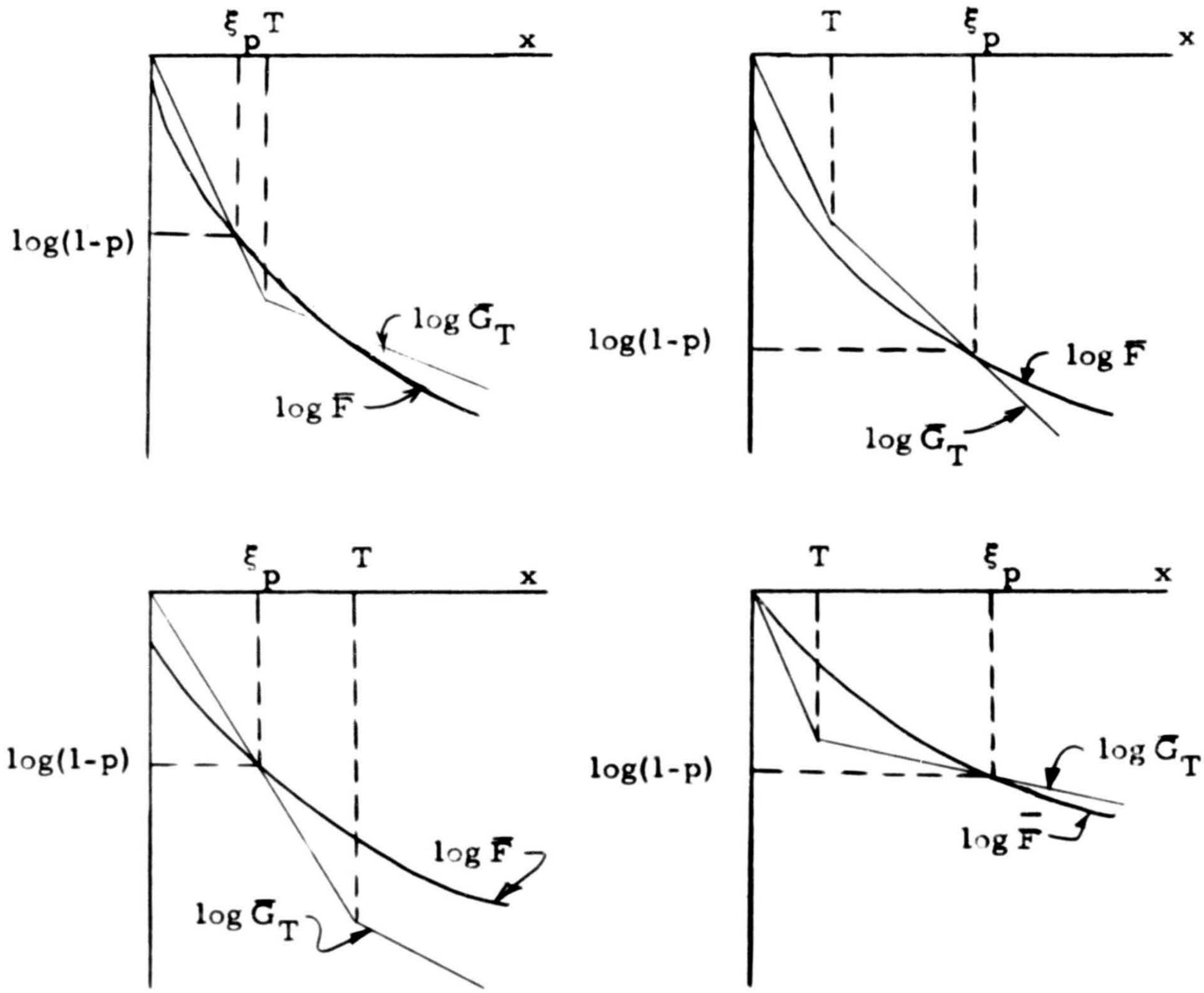
Hence $\quad b_2 = \exp\left(\frac{T}{\xi_p}\log(1-p)\right)\left\{1 + \frac{1-\exp\left(\frac{T}{\xi_p}\log(1-p)\right)}{\xi_p^{-1}\log(1-p)}\right\}^{-1}$

which has a solution continuous in T for all $T > \xi_p$.

Since $1-p \leq e^{-\xi_p}$ and $\mu_1 = 1$ it can clearly be seen that $b_2 \leq b_1$. ‖

Note that $\lim\limits_{T\to 0} \bar{G}_T(x) = \alpha e^{-\alpha x}$ where α is defined uniquely by (4.5). This limit will be denoted by $\bar{G}_0(x)$.

Use is made in the following theorems of the fact that G_T and F must cross at least once if they have the same first moment. The various possibilities for the intertwining of $\log \bar{G}_T$ and $\log \bar{F}$ are shown below.



<u>Theorem 4.2:</u> If F is DFR and has first moment $\mu_1 = 1$ and p^{th} percentile ξ_p given, and also $1 - p < e^{-\xi_p}$, then

$$\bar{F}(t) \leq \begin{cases} e^{-b_1 t} & , \quad 0 \leq t \leq \xi_p \\ \alpha e^{-\alpha t} & , \quad t > \xi_p \end{cases}$$

and the inequalities are sharp. b_1 is defined by equation (4.10) and α by equation (4.5).

Proof:

$0 \le t \le \xi_p$: From the log convexity of $\bar{F}(x)$

$$\bar{F}(t) \le \bar{G}_{\xi_p}(t) \quad , \qquad 0 \le t \le \xi_p$$

$t > \xi_p$: From theorem 4.1,

$$\bar{F}(t) \le \bar{G}_0(t) \quad , \qquad t > \xi_p \ . \ \|$$

Theorem 4.3: If F is DFR and has $\mu_1 = 1$ and p^{th} percentile ξ_p given and also $1 - p < e^{-\xi_p}$; then

$$\bar{F}(t) \ge \begin{cases} \alpha e^{-\alpha t} & , \quad 0 \le t \le \xi_p \\ e^{-b_1 t} & , \quad t > \xi_p \end{cases}$$

where b_1 is defired by (4.10) and α by (4.5). The bound is sharp.

Proof:

$0 \le t \le \xi_p$: This bound follows directly from theorem 4.1.

$\xi_p < t \le \infty$: The proof is obvious. The bound is "epsilon" attained by $\lim\limits_{T \to \infty} \bar{G}_T(x)$. $\|$

Theorem 4.4: If F is DFR , $\mu_1 = 1$ and the p^{th} percentile ξ_p given and also $1 - p < e^{-\xi_p}$; then

$$\frac{\int_t^\infty \bar{F}(x)\,dx}{\bar{F}(t)} \ge \begin{cases} (1 - \frac{1}{\bar{b}_1}) e^{b_1 t} + \frac{1}{\bar{b}_1} & , \quad t \le \xi_p \\ \frac{1}{\bar{b}_2} & , \quad t \ge \xi_p \end{cases}$$

where b_1 is defined by equation (4.10) and b_2 is defined by the equations (4.7) and (4.10) with $T = \xi_p$.

Proof:

For $t \leq \xi_p$: Since $\bar{F}(x) \leq \bar{G}_{\xi_p}(x)$, $x \leq \xi_p$

$$\int_t^{\infty} \bar{F}(x)\,dx \geq \int_t^{\infty} \bar{G}_{\xi_p}(x)\,dx \quad , \quad t \leq \xi_p$$

therefore

$$\frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \geq \frac{\int_t^{\infty} \bar{G}_{\xi_p}(x)\,dx}{\bar{G}_{\xi_p}(t)} = (1 - \frac{1}{b_1})\,e^{b_1 t} + \frac{1}{b_1}$$

For $t > \xi_p$: The proof will be treated in two cases based on the crossings of $\bar{G}_{\xi_p}(x)$ and $\bar{F}(x)$ for $x \geq \xi_p$.

Case (i): Consider the extremal distribution $\bar{G}_{\xi_p}(x)$ and let it cross $\bar{F}(x)$ at $u(\xi_p) > \xi_p$. Obviously due to log convexity the crossing is from above.

Since $\bar{F}(x) \leq \bar{G}_{\xi_p}(x)$ $\qquad x \leq u(\xi_p)$

$$\frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \geq \frac{\int_t^{\infty} \bar{G}_{\xi_p}(x)\,dx}{\bar{G}_{\xi_p}(t)} \qquad t \leq u(\xi_p)$$

Now by Theorem 4.1 $\bar{F}(x) \leq \bar{G}_0(x)$ for $x \geq \xi_p$. Thus due to log convexity of $\bar{F}(x)$, for every x , $u(\xi_p) \leq x \leq \infty$, it is always possible to find a value T , $0 \leq T \leq \xi_p$, such that $u(T) = x$.

$$\therefore \quad \frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \geq \inf_{T \leq \xi_p} \frac{\int_t^{\infty} \bar{G}_T(x)\,dx}{\bar{G}_T(t)} \quad , \quad t \geq u(\xi_p)$$

Case (ii): $u(\xi_p) = \xi_p$. By a similar reasoning to case (i) it may be shown

$$\frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \geq \inf_{T \leq \xi_p} \frac{\int_t^{\infty} \bar{G}_T(x)\,dx}{\bar{G}_T(t)} \quad , \quad t \geq \xi_p$$

$$= \inf_{T \leq \xi_p} \frac{1}{\bar{b}_2}$$

$$= \frac{1}{b_2^*} \quad \text{where } b_2^* \text{ is the value of } b_2$$

calculated at $T = \xi_p$. ||

Thus, the lower bound for the mean residual life while not trivial for $t \geq \xi_p$ is nevertheless not dependent on t and so can give no information on the merit of continued burn-in.

Theorem 4.5: If F is DFR and $\mu_1 = 1$ and the p^{th} percentile ξ_p is given and also $1 - p < e^{-\xi_p}$; then

$$\frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \leq \frac{\int_t^{\infty} \bar{G}_0(x)\,dx}{\bar{G}_0(t)} = \frac{1}{\alpha} \quad , \quad t \leq \xi_p$$

Proof: The proof follows directly from theorem 4.3. ||

Theorem 4.6: If F is DFR with first moment $\mu_1 = 1$ and p^{th} percentile ξ_p, and $1 - p \leq e^{-\xi_p}$, and has hazard rate q then, $q(t^+) \geq \alpha$, where α is the solution of (4.5).

The bound is sharp.

Proof: The proof follows directly from Theorem 4.1. ||

Theorem 4.7: If F is DFR with $\mu_1 = 1$ and p^{th} percentile ξ_p and $1 - p < e^{-\xi_p}$ and hazard rate q then

$$q(t^-) \leq \begin{cases} \infty , & t = 0 \\ b_1 , & t > 0 \end{cases}$$

where b_1 is the solution of (4.7) and (4.8) for $T = t$, $t < \xi_p$; and b_1 is the solution of (4.10) for $t \geq \xi_p$. The bound is sharp.

Proof: $t = 0$ The bound is trivial and is attained by the distribution $\bar{G}_0$.

$0 < t \leq \xi_p$: By the log convexity of the DFR distribution it can be seen that at t

$$q(t^-) \leq b_1$$

$t \geq \xi_p$ Clearly by log convexity

$$q(t^-) \leq b_1 .$$

The bound is relatively trivial and is "epsilon attained" by the distribution $\lim_{T \to \infty} \bar{G}_T$. ||

4.2. Bounds Assuming the First Two Moments Are Given.

Barlow and Marshall (1964) have given bounds on the survival probability of the DFR distribution when the first two moments are given. The same methods used to bound the survival probability may be used to obtain sharp bounds on the residual mean life, the failure rate and the density of the distribution. These bounds are stated below and proofs for them may be found in Lawrence (1964).

$$\mu_t = \frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \geq \mu_1 \tag{4.11}$$

$$\mu_t = \frac{\int_t^{\infty} \bar{F}(x)\,dx}{\bar{F}(t)} \leq \frac{1}{a_2^*} \tag{4.12}$$

$$q(t^-) \leq a_1^* \tag{4.13}$$

$$q(t^+) \geq a_2^* \tag{4.14}$$

$$f(t^-) \leq \begin{cases} \infty & , \quad t = 0 \\ \sup\limits_{T \geq t} a_1 \exp(-a_1 t) & , 0 < t \leq 1 \\ \max \left\{ \begin{array}{ll} \sup\limits_{\frac{2}{\mu_2} \leq \alpha \leq 1} & \alpha^2 \exp(-\alpha\, t) \\ \sup\limits_{T \geq t} & a_1 \exp(-a_1 t) \\ \sup\limits_{T \leq t} & a_2 \exp\{-a_2 t + (a_2 - a_1)T\} \end{array} \right\} & , 1 \leq t \leq \frac{\mu_2}{2} \\ \max \left\{ \begin{array}{ll} \sup\limits_{\frac{2}{\mu_2} \leq \alpha \leq 1} & \alpha^2 \exp(-\alpha t) \\ \sup\limits_{T \leq t} & a_2 \exp\{-a_2 t + (a_2 - a_1)T\} \end{array} \right\} & , \quad t > \frac{\mu_2}{2} \end{cases} \tag{4.15}$$

$$
(4.16)\quad f(t^+) \geq \begin{cases} \min \left\{ \begin{array}{l} \inf\limits_{\frac{2}{\mu_2} \leq \alpha \leq 1} \alpha^2 \exp(-\alpha t) \\[2ex] \inf\limits_{T \leq t} a_2 \exp\{-a_2 t + (a_2 - a_1) T\} \end{array} \right\}, & 0 \leq t \leq \frac{\mu_2}{2} \\[4ex] \inf\limits_{T \leq t} a_2 \exp\{-a_2 t + (a_2 - a_1) T\} , & t \geq \frac{\mu_2}{2} \end{cases}
$$

Where a_1^* , a_2^* are the unique solutions of (4.17) and (4.18) with $T = t$.

$$
(4.17)\qquad 1 = a_1^{-1}(1 - \exp(-a_1 T)) + a_2^{-1} \exp(-a_1 T)
$$

$$
(4.18)\qquad \frac{\mu_2}{2} = a_1^{-2}\{1 - (a_1 T + 1)\exp(-a_1 T)\} + a_2^{-2}(a_2 T + 1)\exp(-a_1 T) \cdot
$$

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